

A Knotty Problem



Book 7 in The Math Kids Series

The Math Kids Series

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1. The Prime-Time Burglars
2. A Sequence of Events
3. An Unusual Pattern
4. An Encrypted Clue
5. An Incorrect Solution
6. The Triangle Secret
7. A Knotty Problem
8. An Artificial Test



A Knotty Problem



Book 7 in The Math Kids Series

by

David Cole



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*To Maude Carmichael,
who inspired my love of math
in the third grade.*



Prologue



The sleek blue helicopter swooped in low over the top of McNair Elementary School. From the helicopter's window, Jordan Waters could clearly see a partially deflated volleyball near one edge of the roof.

"Do you see that?" he asked.

"Yeah, I bet that's the one Dylan kicked up there during recess last week," Justin Grant said.

Jordan started to ask if the pilot could land on the roof so they could retrieve the ball but thought better about it.

"Is that Vivie and Ally on the playground?" Catherine Duchesne asked. The two sisters were twins and never went anywhere without the other.

"I think so," Stephanie Lewis said. She waved her hand frantically as the two girls looked up. The twins waved back and stared as the helicopter settled gently onto the grassy soccer field.

"Thanks so much for the helicopter ride!" Jordan said enthusiastically with an ear-to-ear grin on his face.

David Cole

"Don't mention it, Jordan," Willard Howell said with a smile almost as wide. "It was the least I could do."

The four friends had helped the eccentric billionaire out of a tricky situation. To return the favor, Howell had agreed to grant Jordan's wish of a ride in his helicopter.

Once the rotors had spun down, Howell hopped out of the helicopter and helped the four to the ground.

"You're okay making it home from here?" he asked.

"Sure, Mr. Howell," Justin answered. "It's only a couple of blocks."

"Well, I want to thank each of you again," he said, solemnly shaking each of their hands. "You know I still owe you one, right?"

"We'll keep that in mind, Mr. Howell," Jordan said.

"I mean it. I'm only a phone call away," Howell said. He glanced down at his watch. "I've got to get going now. Thanks again!"

Howell climbed back into the helicopter. The pilot waited until the kids were a safe distance away before spinning up the rotors. The helicopter rose gracefully in the air, pivoted into the wind, and then sped away. The four stared after it until it was just a blue speck in the distance.

"Is that your helicopter?" Vivie asked. She and her sister had come up behind them as the helicopter flew away.

Stephanie laughed. "No, Ally, that's not ours."

"I'm Vivie."

A Knotty Problem

"Are you sure?" Stephanie asked. She was convinced she had finally figured out how to tell the twins apart.

"I'm pretty sure," Vivie said.

"Yeah," Ally added, "she's definitely Vivie."

"Sorry about that," Stephanie said.

"It's okay. It happens all the time," Ally said. "Even our parents get it wrong sometimes."

"Are you sure that's not your helicopter?" Vivie asked again.

"I'm afraid not," Jordan said. "Maybe someday, but not today."

"If you ever get one, will you take us for a ride?" Ally asked.

"Sure," Jordan answered.

"Okay," Ally said. "We've got to go." She and her sister turned and walked away.

"I've got to get home myself," Justin said.

"Me too," Stephanie agreed.

"Okay, but we're still meeting on Saturday after Stephanie's soccer game, right?" Jordan asked. "The district math competition is coming up and we need to be prepared."

"Oh, we'll be ready," Catherine said. "Nothing can stand in our way this year."

She smiled, not knowing their team and their very friendships would soon be tested to the breaking point.

Chapter 3



Justin's premonition was right. Monday morning in class was about as uncomfortable as it could get. Catherine and Stephanie chatted with each other like nothing had happened, but neither had anything to say to Jordan and Justin, who sat just in front of them. There was stiff conversation between the two boys, even some laughter at one of Jordan's bad jokes, but it was clearly forced.

When it was time to break into math groups, things got even more awkward. Normally, the four friends would dive into a new problem with vigor, using teamwork to quickly come up with a solution. Today, without a word, they broke into separate teams. Jordan looked over at Catherine, who shrugged her shoulders in defeat. From that point on, it became clear that the two pairs were in direct competition to solve the latest problem from Mr. Miller.

A Knotty Problem

Use exactly four 4s to form every integer from 0 to 20, using only the following:

+, **-**, **×**, **÷**

Parentheses

Decimal point

√ (Square root)

! (Factorial)

Example: $0 = 4 + 4 - 4 - 4$

"Okay, Justin, we've got this," Jordan said.

Justin gave an uneasy glance toward Stephanie and Catherine. They had their heads down and were already starting to work through the solution.

"Zero is already done, so only twenty numbers to go," Justin said. "Let's do this!"

Wait! Do you want to try this problem before the Math Kids do it?

Use only the four mathematical operations (+, -, ×, ÷), parentheses, decimal points, square roots (√), and factorials (!) and exactly four 4s to form every integer from 0 to 20.

If you need a refresher on the order of operations (what order to do calculations), you can refer to the appendix.

The first few were easy. Jordan and Justin quickly came up with solutions for 1 through 3.

$$1 = 44 \div 44$$

$$2 = 4 \div 4 + 4 \div 4$$

$$3 = (4 + 4 + 4) \div 4$$

The solution for 4 was a little trickier until Justin remembered that anything multiplied by 0 is just 0. That made the solution pretty easy.

$$4 = 4 \times (4 - 4) + 4$$

5 was also a snap.

$$5 = (4 \times 4 + 4) \div 4$$

"This is going to be easy," Jordan said.

Ten minutes later, Justin said, "I think you jinxed us." The solution for 6 had still not come to them. They could do it with three 4s ($4 + 4 \div \sqrt{4}$), but not with four of them. After several more minutes, they decided to go on to the next number.

"Got it!" Justin said. "44 divided by 4 gets us 11, then we just subtract the last 4."

$$7 = 44 \div 4 - 4$$

A Knotty Problem

"Nice!" Jordan said.

In the meantime, Stephanie and Catherine were also making fast work of the first ten numbers.

Stephanie made use of the square root operator to take care of 8.

$$8 = \sqrt{4} + \sqrt{4} + \sqrt{4} + \sqrt{4}$$

Catherine came up with a tricky way to solve for 9.

$$9 = 44 \div 4 - \sqrt{4}$$

"Nice work!" Stephanie exclaimed. "I had another way to do it, but I like yours better."

Both girls came up with a solution for 10 at the same time and they laughed as both wrote down the same answer.

$$10 = 4 + 4 + 4 - \sqrt{4}$$

"Halfway there," Catherine said. "How are you guys doing?"

Both Justin and Jordan looked up at her question.

"Same here," Justin said. "We've got all but a solution for the number 6."

"Really?" Stephanie asked. "That one was really easy." She looked down at the paper. "In fact, we've got four different solutions for that one."

"You do not!" Justin said, trying to look over at the girls' solution page.

"Trying to copy, Justin?" Catherine asked.

"No, I just don't believe you have four different ways for 6."

"Really?" Stephanie asked. "How about this one to start?" She showed him her equation:

$$4 + \sqrt{4} \times 4 \div 4$$

"That's only one," Justin shot back.

"Okay, how about $4 + (4 + 4) \div 4$?" Catherine asked. It was clear she was enjoying the fact the girls had multiple solutions for a problem the guys couldn't solve.

"And $(4 + 4 + 4) \div \sqrt{4}$," Stephanie said.

"And $\sqrt{4} + \sqrt{4} + 4 \div \sqrt{4}$," Catherine added.

"Now you're just showing off," Justin grumbled.

"It sounds like someone is a little jealous of our amazing math skills," Stephanie responded. Catherine giggled.

"We'll just see who gets to the end first," Jordan said determinedly.

He and Justin doubled their efforts and quickly found solutions for the next two numbers.

$$11 = 44 \div (\sqrt{4} + \sqrt{4})$$

$$12 = (44 + 4) \div 4$$

A Knotty Problem

They stumbled when they got to 13, so they decided to move on to 14, which Justin solved.

$$14 = 4 \times 4 - (4 \div \sqrt{4})$$

Justin's solution for 14 gave Jordan the inspiration for his answer for 15.

$$15 = 4 \times 4 - 4 \div 4$$

16 was a no brainer for the two, with both Justin and Jordan coming up with the easy answer.

$$16 = 4 + 4 + 4 + 4$$

Justin glanced over at the girls. It looked like they had also been stumped on a solution for 13 and had moved on to 17, putting them in a tie with him and Jordan.

"We've got to hurry, Jordan," he whispered. There was a note of desperation in his voice.

The girls had solved 17 and were now getting close to the end. Catherine had figured out that the answers for 15 and 17 were very similar.

$$17 = 4 \times 4 + 4 \div 4$$

Stephanie figured out 18.

$$18 = 4 \times 4 + 4 \div \sqrt{4}$$

Now they only had two to go and they could go back and tackle the tricky number 13. Except 19 also proved to be difficult. Neither Stephanie nor Catherine was able to quickly come up with a solution. Stephanie glanced at the boys' paper. They had skipped 19 and were working on 20.

Jordan nailed 20 and quickly scratched his answer onto the paper.

$$20 = 4 \times 4 + \sqrt{4} + \sqrt{4}$$

Catherine came up with a different answer.

$$20 = 4 \times (4 + 4 \div 4)$$

Both teams were now down to the same problems to solve: 13 and 19. It would be a race to the finish! Catherine was quickly trying various solutions. Jordan was scribbling on a piece of scratch paper, his pencil worn down to a small stub. Stephanie was examining previous answers to see if there was a similar method she could use for the last two problems. Justin was doing none of these things. He was just looking across the room.

Those who didn't know Justin would have thought he had become entranced with the bulletin board Mr. Miller had put up to illustrate the parts of speech. But when

A Knotty Problem

Jordan glanced over at his friend and saw him staring intently at the bulletin board, he smiled. He knew Justin was "in the zone," the place he went when he was trying to come up with a new idea. When Justin was in the zone, his mind was so focused that he lost track of everything else. One summer he had been thinking so hard about a quicker way to make his bed that he had walked right into the swimming pool. He had come up sputtering in the three-foot-deep water, but his plan of tying strings to the corners of the sheet and bedspread and pulling them to the head of the bed using a pulley system had worked rather well. Now Jordan was hoping he would come out of the zone with something that would allow them to finish the problem ahead of Stephanie and Catherine. He was not disappointed.

"Factorials!" Justin said, blinking his eyes to regain focus. "4 factorial is 24."

And that was all it took. Unfortunately, he had said it so loudly that now the girls knew as well. The race was on!

"Got 19!" yelled Catherine.

$$19 = 4! - 4 - 4 \div 4$$

"Me too!" said Justin just seconds later.

In the end, both teams approached the final answer in the same way. Knowing that 4 factorial was 24, they

knew they had to find a way to subtract 11 to get to the final answer of 13. They scanned their previous solution for 11 and two hands scratched out a solution at the same time.

"Done!" Justin and Stephanie said in unison, scribbling their answers onto the paper.

$$13 = 4! - 44 \div 4$$

"We were first," Justin said.

"It was a tie," Stephanie replied.

"No way," Justin argued. "I definitely said it first."

"Well, I wrote it down first, so we won."

"Did not!"

"Did so!"

"What is going on over here?" came the gruff voice of Mr. Miller. "Is there some kind of problem?"

"We were racing to see who could answer the problem first and we won," Justin answered.

"Except you didn't," Stephanie said.

"I said it first," Justin said.

"I wrote it down first."

"No, you didn't," Justin countered.

"It was close," Jordan said, trying to calm them both. "Why don't we call it a tie?"

"It doesn't matter anyway," Stephanie said. "You guys needed help to get the number 6, so you wouldn't have even finished without us." She gave Justin a smug smile.

A Knotty Problem



"I'm with Jordan," Catherine said. "Let's call it a tie."

"But we won," Stephanie protested.

"I thought you four were a team," Mr. Miller said.

"Not anymore," Stephanie said. "Justin kicked Catherine and me out of the Math Kids."

David Cole

"No, I didn't," Justin fumed. "You both quit."

"Only because you didn't give us a choice," Stephanie said. "You told us—"

"That's quite enough, you two," Mr. Miller interrupted. "Let's all return to our seats and we can discuss this later."

Appendix



Order of Operations

In math, the order of operations is a set of rules that tell us the sequence in which the operations should be performed to solve an equation.

The order is:

1. **P**arentheses
2. **E**xponents and **R**oots
3. **M**ultiplication/**D**ivision
4. **A**ddition/**S**ubtraction

In the United States, this order is sometimes referred to as PEMDAS. Another way of remembering the order is by using the mnemonic “**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally.”

Note that in some countries, the acronym used for the order of operations may be slightly different. In Canada and New Zealand, for example, the order of operations is

referred to as BEDMAS (**B**rackets, **E**xponents, **D**ivision/**M**ultiplication, **A**ddition/**S**ubtraction). Other countries use BODMAS (**B**rackets, **O**rders, **D**ivision/**M**ultiplication, **A**ddition/**S**ubtraction).

To use the order of operations, you would first do anything that is in parentheses or brackets, then evaluate any exponents or roots, then do any multiplication or division, then do any addition or subtraction. Note that multiplication and division are at the same level, so if an expression has both multiplication and division, we would evaluate them from left to right. The same holds true for addition and subtraction.

Here is an example:

$$8 + (3 - 2) \times 4^2 \div 2$$

(P) First evaluate what is inside the parentheses.

$$(3 - 2) = 1, \text{ leaving us with } 8 + 1 \times 4^2 \div 2$$

(E) Next, evaluate the exponent.

$$4^2 = 16, \text{ leaving us with } 8 + 1 \times 16 \div 2$$

(MD) Next, do the multiplication and division from left to right.

$$8 + 1 \times 16 \div 2$$

$$8 + 16 \div 2$$

$$8 + 8$$

(AS) Finally, do the addition and subtraction from left to right.

$$8 + 8 = 16$$

A Knotty Problem

What would happen if we ignored PEMDAS and just did everything from left to right?

$$8 + (3 - 2) \times 4^2 \div 2$$

$$8 + 3 \quad \rightarrow \quad 11 - 2 \times 4^2 \div 2$$

$$11 - 2 \quad \rightarrow \quad 9 \times 4^2 \div 2$$

$$4^2 \quad \rightarrow \quad 9 \times 16 \div 2$$

$$9 \times 16 \quad \rightarrow \quad 144 \div 2$$

$$144 \div 2 \quad \rightarrow \quad 72$$

We would get a very different answer! That's why it's important that we follow these rules so that everyone will get a consistent answer to the same expression.

Fermat's Last Theorem

When Catherine was addressing the school board, she mentioned Andrew Wiles and his proof of Fermat's Last Theorem. A theorem is a statement which has been proved by a logical argument called a proof.

Fermat's Last Theorem says that there are no positive integers a , b , and c that satisfy the equation

$$a^n + b^n = c^n, \text{ except where } n = 1 \text{ or } 2.$$

What does this mean?

First, a^n means to multiply a by itself n times. For

example, a^3 means to multiply a by itself three times. In other words, $a^3 = a \times a \times a$. In the simplest case, $a^1 = a$.

Fermat's Last Theorem says that we can find three numbers that satisfy $a^1 + b^1 = c^1$. One example is $1 + 2 = 3$.

Easy, right? There are infinite solutions to this.

The theorem also says we can find three numbers that satisfy the equation $a^2 + b^2 = c^2$. One example is $3^2 + 4^2 = 5^2$

But Fermat's Last Theorem says we can't find solutions for $a^3 + b^3 = c^3$ or $a^4 + b^4 = c^4$ or any number greater than 2.

The theorem was first stated by Pierre de Fermat around 1637. It was written in the margin of a book called *Arithmetica*. Fermat said he had a proof, but that it was too large to fit in the margin. It became known as Fermat's Last Theorem because all his other theorems were proved. It was 358 years before anyone was able to prove it. Andrew Wiles, an English mathematician, published a proof in 1995.

Before it was finally proved, Fermat's Last Theorem was listed in the *Guinness Book of World Records* as the "most difficult mathematical problem" because it had so many unsuccessful proofs.

The quote Catherine used when discussing Andrew Wiles was from an interview he did for the NOVA program. You can read that interview to learn more about Andrew Wiles and how he solved Fermat's Last Theorem: www.pbs.org/wgbh/nova/article/andrew-wiles-fermat/.

A Knotty Problem

George Polya

George Polya was a Hungarian mathematician. He taught at ETH Zurich in Switzerland from 1914 to 1940 and at Stanford University from 1940 to 1953. He became known as “the father of problem-solving” for his approach to math education. At the school board meeting, Catherine quotes from his book *How To Solve It*, which has been in print since 1945 and has sold over a million copies.

Gordian Knot

According to Greek legend, the story of Alexander the Great being challenged by the Gordian knot is true, although there are different versions of how he solved the problem. The popular account, as told by Mr. Duchesne in the book, is that he used his sword to cut the knot in half. Some classical scholars, however, believe it is more likely that he pulled the knot out of the pole pin, which exposed the two ends of the cord. Once he had the two ends free, he could untie the knot.

District Math Competition Head-to-Head Questions

1. *If five negative numbers are multiplied together, will the product be positive or negative?*

When two positive numbers are multiplied together, the product will always be a positive number. When two negative numbers are multiplied together, the product will also be a positive number. The only time the product will be negative is if a positive number is multiplied by a negative number.

If there are an even number of negative numbers, the product will always be a positive number (two negatives multiplied together will be positive). If there are an odd number of negative numbers, the product will always be a negative number.

2. *The sum of the heights of Paul and Rachel is ninety-four inches. Rachel is eight inches taller than Paul. How tall is Paul?*

Let's use P to represent Paul's height and R to represent Rachel's height. We know the sum of their heights is 94 inches. This gives us the equation

$$P + R = 94$$

We also know Rachel is 8 inches taller than Paul. If we write this as an equation, we get

$$R = P + 8$$

A Knotty Problem

We can substitute the second equation into the first equation (instead of using R , we'll use $P + 8$). This gives us

$$P + (P + 8) = 94$$

Another way of writing this is

$$2P + 8 = 94$$

If we subtract 8 from both sides of the equation, we have

$$2P = 76$$

If we divide both sides of the equation by 2, we're left with

$$P = 38$$

So Paul is 38 inches tall.

Let's check our answer. Since Rachel is 8 inches taller than Paul, that means Rachel is $38 + 8$ or 46 inches tall. That means the sum of their heights is $38 + 46 = 94$ inches, which checks out!

3. *A math test has ten problems. Five points are given for each correct answer and two points deducted for each incorrect answer. Roshan answered all ten questions and scored twenty-nine points. How many correct answers did he have?*

Let's use C to represent a correct answer and W to represent a wrong answer. We know the total number of answers was 10, so we have

$$C + W = 10 \text{ (another way to write this is } W = 10 - C)$$

We also know that Roshan scored a total of 29 points. If there are +5 points for every correct answer and -2 points for every wrong answer, that means

$$5C - 2W = 29$$

If we substitute $(10 - C)$ for W in this equation, we get

$$5C - 2(10 - C) = 29$$

To multiply -2 by $(10 - C)$, we first multiply -2 by 10 to get -20 . Then we multiply -2 by $-C$ to get $2C$ (remember that multiplying two negative numbers will give us a positive product). Now we have

$$5C - 20 + 2C = 29$$

If we combine the C s together, we get

$$7C - 20 = 29$$

If we add 20 to both sides of the equal sign, we get

$$7C = 49$$

Dividing both sides by 7, we get

$$C = 7$$

That means Roshan got 7 answers right (and 3 answers wrong). He got 35 points (5×7) for his correct answers and lost 6 points (3×2) for his incorrect answers. Since $35 - 6 = 29$, our math checks out!

4. A camera and case cost one hundred dollars. If the camera cost ninety dollars more than the case, how much does the case cost?

A Knotty Problem

The Armstrong team made a common mistake on this problem. They subtracted 90 from 100 to get 10 for the cost of the case. Where did they go wrong?

If we use C to represent the cost of the camera and S to represent the cost of the case, we have

$$C + S = 100$$

If the camera cost 90 dollars, the case would be 10 dollars. But the problem says the camera cost 90 dollars more than the case.

If we write this down as an equation, it means

$$C = S + 90$$

Let's substitute $(S + 90)$ for C in our first equation.

We get

$$(S + 90) + S = 100$$

Another way of writing this is

$$2S + 90 = 100$$

After subtracting 90 from both sides, we have

$$2S = 10$$

Dividing both sides by 2, we find

$$S = 5$$

So the cost of the case is 5 dollars, not 10 like the Armstrong team answered.

Since the camera is 90 dollars more than the case, the camera cost 95 dollars. If we add 95 (the cost of the camera) to 5 (the cost of the case), we get 100 dollars, so our math checks out.

5. Six dollars were exchanged for nickels and dimes. The number of nickels was the same as the number of dimes. How many nickels were in the change?

Let N be the number of nickels and D be the number of dimes. That gives us two equations:

$N = D$ There are an equal number of nickels and dimes.

$5N + 10D = 600$ Nickels are 5 cents and dimes are 10 cents. They must total 6 dollars, which is 600 cents.

We can substitute N for D in the second equation to get

$$5N + 10N = 600$$

$$15N = 600$$

$$N = 40$$

That means there are 40 nickels (and 40 dimes). 40 nickels is 2 dollars and 40 dimes is 4 dollars, so they total 6 dollars.

6. There are six three-digit numbers that can be formed using each of the digits 4, 5, and 6 exactly once. What is the average of these six three-digit numbers?

Let's start by finding all the three-digit numbers

A Knotty Problem

that can be formed using the digits 4, 5, and 6 exactly once.

456

465

546

564

645

654

We could calculate the average by simply adding these numbers together and dividing by 6 (the number of numbers), but Catherine answered this question very quickly in the story. Did she find a way to make it easier?

Look at the first column of numbers (the hundreds digit). You will notice there are two 4s, two 5s, and two 6s. Now look at the second column (the tens digit). Same story: two 4s, two 5s, and two 6s. Finally, look at the third column (the ones digit). Again, we'll find two 4s, two 5s, and two 6s.

If we average two 4s, two 5s, and two 6s, the average is 5. We can check this out by adding the numbers and dividing by 6.

$$(4 + 4 + 5 + 5 + 6 + 6) = 30 \text{ and } 30 \div 6 = 5$$

Catherine recognized that each column would average to 5, so she was able to quickly give the final answer of 555.

7. One loaf of bread and six rolls cost one dollar and eighty cents. Two loaves of bread and four rolls cost two dollars and forty cents. How much does one loaf of bread cost?

Let B be the cost of a loaf of bread and R be the cost of a roll.

$$B + 6R = 180 \text{ (since \$1.80 is the same as 180 cents)}$$

Another way to write this equation is

$$B = 180 - 6R$$

We know that 2 loaves of bread and 4 rolls cost \$2.40, or

$$2B + 4R = 240$$

Since we know $B = 180 - 6R$, we can substitute for B to get

$$2(180 - 6R) + 4R = 240$$

$$360 - 12R + 4R = 240$$

$$360 - 8R = 240$$

Adding $8R$ to both sides, we get

$$360 = 240 + 8R$$

Subtracting 240 from both sides, we get

$$120 = 8R$$

That means that each roll costs 15 cents ($120 \div 8$).

Since $B = 180 - 6R$, we can substitute for R to get

$$B = 180 - 90 = 90$$

Each loaf of bread is 90 cents.

A Knotty Problem

8. *There are four large boxes. Inside each large box are three medium boxes. In each medium box, there are two small boxes. How many total boxes are there?*

There are three sizes of boxes. We can call them L (large), M (medium), and S (small).

We know there are 4 large boxes, so $L = 4$.

In each large box, there are 3 medium boxes, so $M = 4 \times 3 = 12$.

In each medium box, there are 2 small boxes, so $S = 12 \times 2 = 24$.

Now we just add $L + M + S$ to get the total number of boxes.

$$\begin{aligned}\text{Total boxes} &= L + M + S \\ &= 4 + 12 + 24 \\ &= 40\end{aligned}$$

9. *A baseball league has nine teams. During the season, each team plays three games with each of the other teams. What is the total number of games played?*

Each team plays 3 games each against the 8 other teams in the league. Since there are 9 teams, we just multiply $9 \times 3 \times 8 = 216$.

That was the answer given by Armstrong, but it wasn't right. Why? In each game, there are two teams playing. Stephanie recognized this, so she divided 216 by 2 to get the actual number of games played. Her answer of 108 was correct.

10. *The average of five numbers is six. If one of the numbers is removed, the average of the remaining four numbers is seven. What number was removed?*

Catherine was able to quickly answer this question. How did she do it so fast?

How do we calculate the average of a group of numbers? We first add up all the numbers to get the sum. Then we divide this by the size of the group.

$$\text{Average} = (\text{sum of numbers}) \div (\text{size of group})$$

There's another way we can look at this equation. If we know the average and the size of the group (the number of numbers), we can easily calculate the sum.

$$\text{Sum} = \text{Average} \times \text{size of group}$$

Catherine quickly figured out the sum of the numbers by multiplying the average (6) by the number of numbers (5) to get 30.

Now a number is removed from the group. The new average is seven. Catherine calculated the new sum.

$$\text{New sum} = \text{new average} \times \text{new size of group}$$

$$\text{New sum} = 7 \times 4$$

$$28 = 7 \times 4$$

Since the original sum was 30 and the new sum is 28, the number removed had to have been the number 2.

Coming Next!

An Artificial Test

Book 8 in The Math Kids Series

by

David Cole

